Pearson Edexcel

Mark Scheme (Results)

October 2024

Pearson Edexcel International Advanced Level
In Pure Mathematics (WMA13) Paper 01

Question Number	Scheme	Marks
1	$3\tan^2\theta+7\sec\theta-3=0\Rightarrow 3\left(\sec^2\theta-1\right)+7\sec\theta-3=0$	M1
	$3\sec^2\theta+7\sec\theta-6=0$	A1
	$(3\sec\theta-2)(\sec\theta+3)=0\Rightarrow \sec\theta=...\Rightarrow\cos\theta=...$	dM1
	$\theta=109.5°, 250.5°$	A1, A1
		(5)
		Total 5

Main method: Also allow if another variable is used e.g $x\leftrightarrow\theta$

M1: Attempts to set up a quadratic equation in $\sec\theta$ using $\tan^2\theta=\pm1\pm\sec^2\theta$

Condone bracketing slips, for example, where $3\tan^2\theta$ is replaced by $\pm3\sec^2\theta\pm1$

A1: Collects terms to form a correct 3TQ, $3\sec^2\theta+7\sec\theta-6=0$ o.e

Terms do not need to be all on one side of the equation and the $=0$ can be implied by subsequent work.

dM1: Dependent upon the previous M1, it is scored for the method of producing a solvable non zero solution for $\cos\theta$

Look for;

- A valid way of solving their quadratic equation in $\sec\theta$ using any allowable method including a calculator leading to $\sec\theta=...$
- Use of $\sec\theta=\frac{1}{\cos\theta}$ to produce a solution $\cos\theta=k$ where $-1<k<1, k\neq0$

E.g. $3\sec^2\theta+7\sec\theta-6=0\Rightarrow\sec\theta=-3\Rightarrow\cos\theta=-\frac{1}{3}$. You can ignore the 2nd solution

If the error had been $\tan^2\theta=\sec^2\theta+1$ look for $3\sec^2\theta+7\sec\theta=0\Rightarrow\sec\theta=-\frac{7}{3}\Rightarrow\cos\theta=-\frac{3}{7}$

A1: One of $\theta=$ awrt $109.5°$, awrt $250.5°$ following M1, A1, dM1

Allow either answer in radians here so score for awrt 1.91 or awrt 4.37

A1: CSO. Both $\theta=$ awrt $109.5°$ and awrt $250.5°$ and no extras in the range

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Other methods do exist but the same marking principles will apply

Alt method:

M1: For an attempt to set up a quadratic equation in a single trigonometric term, e.g. $\cos\theta$

Replaces $\tan^2\theta$ by $\frac{\sin^2\theta}{\cos^2\theta}$, $\sec\theta$ by $\frac{1}{\cos\theta}$ and then uses $\sin^2\theta=\pm1\pm\cos^2\theta\Rightarrow$ quadratic equation in $\cos\theta$ E.g. $3\tan^2\theta+7\sec\theta-3=0\Rightarrow3\frac{\sin^2\theta}{\cos^2\theta}+7\times\frac{1}{\cos\theta}-3=0\Rightarrow3\sin^2\theta+7\cos\theta-3\cos^2\theta=0$

and then uses $\sin^2\theta=\pm1\pm\cos^2\theta\Rightarrow3\left(1-\cos^2\theta\right)+7\cos\theta-3\cos^2\theta=0$.

Condone slips

A1: For a correct 3TQ in $\cos\theta$. FYI $6\cos^2\theta-7\cos\theta-3=0$. Terms do not have to be on one side

dM1: Solves the quadratic equation in $\cos\theta$ using an appropriate method including a calculator.

It must lead to a solution $\cos\theta=k$ where $-1<k<1, k\neq0$

If a calculator is used the solution must be correct for their equation.

E.g. $6\cos^2\theta - 7\cos\theta - 3 = 0 \Rightarrow \cos\theta = -\frac{1}{3}$

A1: One of $\theta =$ awrt $109.5°$, awrt $250.5°$ following M1, A1, dM1

Allow either answer in radians here so score for awrt 1.91 or awrt 4.37

A1: CSO. Both $\theta =$ awrt $109.5°$ and awrt $250.5°$ and no extras in the range.

If both solutions to the quadratic were given then both must be correct

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Methods using calculator technology and not showing all stages of working as required by question:

Example 1:

$$3\tan^2\theta + 7\sec\theta - 3 = 0 \Rightarrow \theta = 109.5°, 250.5°$$

No marks

Example 2:

$$3\tan^2\theta + 7\sec\theta - 3 = 0 \Rightarrow 3\left(\sec^2\theta - 1\right) + 7\sec\theta - 3 = 0$$

$$\Rightarrow 3\sec^2\theta + 7\sec\theta - 6 = 0 \Rightarrow \sec\theta = -3 \Rightarrow \theta = 109.5°, 250.5°$$

Scores M1, A1, dM0, A0, A0

Example 3:

$3\tan^2\theta + 7\sec\theta - 3 = 0 \Rightarrow 6\cos^2\theta - 7\cos\theta - 3 = 0$ via identities followed by $\Rightarrow \theta = 109.5°, 250.5°$

Scores M1, A1, dM0, A0, A0

Methods using calculator technology and showing sufficient stages of working:

Example 4: $3\tan^2\theta + 7\sec\theta - 3 = 0 \Rightarrow 3\sec^2\theta + 7\sec\theta - 6 = 0$ via identity followed by

$$\sec\theta = -3 \Rightarrow \cos\theta = -\frac{1}{3} \Rightarrow \theta = \pm 109.5°$$

Scores M1, A1, dM1, A1, A0 (only one of the two solutions given)

Example 5: $3\tan^2\theta + 7\sec\theta - 3 = 0 \Rightarrow 6\cos^2\theta - 7\cos\theta - 3 = 0$ via identities followed by

$\cos\theta = -\frac{1}{3} \Rightarrow \theta = 109.5°, 250.5°$ M1, A1, dM1, A1, A1

Question Number	Scheme	Marks
2 (a)	$x=2y^2+5y-6$	
	$\frac{dx}{dy}=4y+5 \Rightarrow \frac{dy}{dx}=\frac{1}{4y+5}$	M1, A1
		(2)
(b)	Sets their $4y+5=0 \Rightarrow y=-\frac{5}{4}$	M1
	Substitutes their $y=-\frac{5}{4}$ into $x=2y^2+5y-6$ to find x	dM1
	$\left(-\frac{73}{8},-\frac{5}{4}\right)$	A1
		(3)
		Total 5

Mark as one complete question

Main method via differentiation in (a) and (b)

(a)

M1: Differentiates $2y^2+5y-6$ to a linear term in y AND attempts to take the reciprocal.

Condone for this mark attempts such as $\frac{dx}{dy}=4y+5 \Rightarrow \frac{dy}{dx}=\frac{1}{4y}+\frac{1}{5}$

We may see other methods including implicit differentiation from WMA14.

Look for $x=2y^2+5y-6 \Rightarrow 1=ay\frac{dy}{dx}+b\frac{dy}{dx}$ followed by an attempt to make $\frac{dy}{dx}$ the subject

A1: $\frac{dy}{dx}=\frac{1}{4y+5}$ ISW after a correct answer.

(b) **Marks in part (b) can only be awarded after sight of** $\frac{dx}{dy}=\frac{ay+b}{k}$ **or** $\frac{dy}{dx}=\frac{k}{ay+b}$

So full marks can only be awarded after sight of $\frac{dx}{dy}=4y+5$ **or** $\frac{dy}{dx}=\frac{1}{4y+5}$

M1: Sets $'ay+b'=0 \Rightarrow y=....$ following sight of $\frac{dx}{dy}=\frac{ay+b}{k}$ **or** $\frac{dy}{dx}=\frac{k}{ay+b}$

Ignore/condone incorrect statements such as $\frac{dy}{dx}=0$ here

This can be implied. E.g $\frac{dy}{dx}=\frac{1}{4y+5}$ followed by $y=-\frac{5}{4}$

dM1: Substitutes their solution of their $4y+5=0$ into $x=2y^2+5y-6$ to find x.

This is dependent upon the previous M

A1: $\left(-\frac{73}{8},-\frac{5}{4}\right)$ following $\frac{dx}{dy}=4y+5$ **or** $\frac{dy}{dx}=\frac{1}{4y+5}$

The answer may be given separately, for example, as $x=-9.125$, $y=-1.25$

ISW after a correct answer.

Further guidance to highlighted sentence.

If (a) is incorrect or incomplete follow the following guidance.

Incorrect (a): If they incorrectly state $\frac{\mathrm{d}y}{\mathrm{d}x}=4y+5$ o.e. and go on to use $4y+5=0\Rightarrow y=...$ and write down a correct set of coordinates for P they will score NO MARKS in (b)

Incomplete (a): If they state $\frac{\mathrm{d}x}{\mathrm{d}y}=4y+5$ in (a) and go on to use $4y+5=0\Rightarrow y=...$ in (b) they will score no marks in (a) but could potentially score full marks in (b)

You can award all 3 marks for $\left(-\frac{73}{8},-\frac{5}{4}\right)$ following either $\frac{\mathrm{d}x}{\mathrm{d}y}=4y+5$ or $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{4y+5}$

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Alternative approach: Part (a) may well be blank or incorrect. Part(b) done via completion of square

It is possible to do part (b) by not doing any differentiating.

$$x=2y^2+5y-6\Rightarrow x=2\left(y+\frac{5}{4}\right)^2-\frac{73}{8}$$

M1: An allowable attempt to complete the square followed by a valid attempt at either x or y from their attempt
A1: For a correct x or y
A1: For a correct x and y
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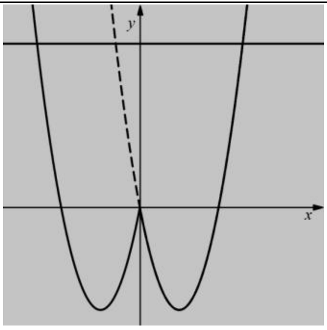
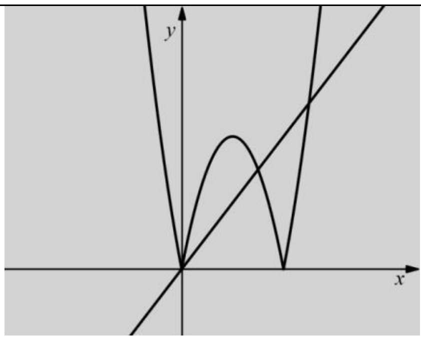
Question Number	Scheme	Marks
3 (a)	 $y = 48$ Graphical interpretation of $\mathrm{f}\left(\lvert x \rvert\right) = 48$	
	One of $8, -8$	B1
	Attempts to solve an appropriate equation E.g. $2x^2 - 10x = 48 \Rightarrow x^2 - 5x - 24 = 0 \Rightarrow (x \pm 8)(x \pm 3) = 0 \Rightarrow x = \ldots$	M1
	$x = 8, -8$ with no additional values	A1
		(3)
(b)	 $y = \frac{5}{2}x$ Graphical interpretation of $\lvert \mathrm{f}(x) \rvert \geqslant \frac{5}{2}x$	
	Attempts to solve $2x^2 - 10x = \frac{5}{2}x \Rightarrow 4x^2 - 25x = 0 \Rightarrow x = \frac{25}{4}$ OR attempts to solve (o.e.) $10x - 2x^2 = \frac{5}{2}x \Rightarrow 4x^2 - 15x = 0 \Rightarrow x = \frac{15}{4}$	M1
	Attempts to solve $2x^2 - 10x = \frac{5}{2}x \Rightarrow 4x^2 - 25x = 0 \Rightarrow x = \frac{25}{4}$ AND attempts to solve (o.e.) $10x - 2x^2 = \frac{5}{2}x \Rightarrow 4x^2 - 15x = 0 \Rightarrow x = \frac{15}{4}$	dM1
	Achieves both critical values $x = \frac{15}{4},\ x = \frac{25}{4}$	A1
	Correct set of values $x \leqslant \frac{15}{4}$ or $x \geqslant \frac{25}{4}$	A1
		(4)
		Total 7

This question demands that some working should be shown

(a) This part is now being marked B1 M1 A1 instead of M1 A1 A1

B1: Correctly obtains 8 or -8 as a solution (for x) . Condone solution appearing as $\lvert x \rvert = 8$

M1: Attempts to solve $\mathrm{f}\left(\lvert x \rvert\right) = 48$ via an appropriate method including a calculator, factors or formula

So, look for a correct equation followed by an attempt to solve.

The equation must be given in the form of a correct 3TQ, but not necessarily collected on one side of the = sign. Solving is allowed by all methods including via a calculator.

All the following score this M mark

Example 1:

$$2x^2 - 10x = 48 \text{ o.e } \Rightarrow x = 8, (-3) \ \ldots\text{only the 8 is required}$$

Example 2:

$$2x^2 - 10x = 48 \text{ o.e } \Rightarrow x^2 - 5x - 24 = 0 \Rightarrow (x-8)(x+3) = 0 \Rightarrow x = ...$$

Example 3:

$$2|x|^2 - 10|x| - 48 = 0 \text{ or } |x|^2 - 5|x| - 24 = 0 \text{ leading to } |x| \text{ or } x = 8, (-3)$$

Example 4:

$$2x^2 + 10x - 48 = 0 \Rightarrow x = \frac{-10 \pm \sqrt{100 - 4(2)(-48)}}{2(2)} = ...$$

Example 5:

$$2x^2 + 10x = 48 \text{ o.e } \Rightarrow x = -8, (3) \ \ldots\text{only the } -8 \text{ is required}$$

A1 $x = 8, -8$ with no additional values unless they have been rejected

(b)

M1: Attempts to solve a correct equation or inequality, leading to a non-zero value for x.
The method **must be shown** with the terms being collected as a minimum response

Look for $2x^2 - 10x = \frac{5}{2}x \Rightarrow ax^2 \pm bx = 0 \Rightarrow x = c, c \neq 0$. Allow with the = as any inequality including <

OR $-2x^2 + 10x = \frac{5}{2}x \Rightarrow ax^2 \pm bx = 0 \Rightarrow x = c, c \neq 0$. Allow with the = as any inequality including <

You may see as an alternative to the 2nd equation/inequality $2x^2 - 10x = -\frac{5}{2}x \Rightarrow ax^2 \pm bx = 0 \Rightarrow x = c, c \neq 0$

It is acceptable to divide through by x, so $2x^2 - 10x = \frac{5}{2}x \Rightarrow 2x - 10 = \frac{5}{2} \Rightarrow 2x = \frac{25}{2} \Rightarrow x = 6.25$

dM1: Attempts to solve BOTH correct equations or inequations leading to two non-zero values for x
Allow even if there are extra equations solved.

A1: Achieves both critical values $x = \frac{15}{4}$ and $x = \frac{25}{4}$ and no other values apart from 0 (and perhaps 5) following both equations or inequalities. Allow with incorrect inequalities $x \geqslant \frac{15}{4}$ and $x \leqslant \frac{25}{4}$, you are looking for the critical values only

A1: Correct range in allowable form. E.g. $x \leqslant \frac{15}{4}$ or $x \geqslant \frac{25}{4}$, $\left(-\infty, \frac{15}{4}\right] \cup \left[\frac{25}{4}, \infty\right)$ but condone $x \leqslant \frac{15}{4}, x \geqslant \frac{25}{4}$

Do NOT incorrect forms such as $x \leqslant \frac{15}{4}$ and $x \geqslant \frac{25}{4}$, $\left(-\infty, \frac{15}{4}\right] \cap \left[\frac{25}{4}, \infty\right)$, $\frac{25}{4} \leqslant x \leqslant \frac{15}{4}$

The two aspects must be brought together on a single line. Mark their final answer

Alt Method: You may see attempts via squaring. It can be marked using the above guidelines

$$\left(2x^2 - 10x\right)^2 = \left(\frac{5}{2}x\right)^2 \Rightarrow x^2\left(16x^2 - 160x + 375\right) = 0 \Rightarrow x^2(4x - 15)(4x - 25) = 0$$

Question Number	Scheme	Marks
4 (a)	$\log_{10} N = 2 \Rightarrow N = 100$	B1
		(1)
(b)	$\log_{10} N = 0.35t + 2 \Rightarrow N = 10^{0.35t+2}$	M1
	$\Rightarrow N = 10^{0.35t} \times 10^{2}$	dM1
	$\Rightarrow N = 100 \times 2.24^{t}$	A1
		(3)
(c)	Rate of growth at 5 hours is $\frac{dN}{dt} = 100 \times \ln 2.24 \times 2.24^{5}$	M1
	Allow an answer in the range 4530 to 4550 (bacteria per hour)	A1
		(2)
		Total 6

(a)

B1: For 100 Do not accept just 10^2 and cannot be awarded from 100 in part (b)

(b) **The demand of the question is 'show that', not just find values of *a* and *b***

Method 1: starting with $\log_{10} N = 0.35t + 2$

M1: Undoes the log. E.g. $\log_{10} N = 0.35t + 2 \Rightarrow N = 10^{0.35t+2}$

Give BOD for those who go from $\log_{10} N = 0.35t + 2$ straight to $N = 10^{0.35t} \times 10^{2}$

dM1: And follows with a correct application of the index law.

Look for $\log_{10} N = 0.35t + 2 \Rightarrow N = 10^{0.35t+2} = 10^{0.35t} \times 10^{2}$ o.e such as $10^{0.35t} \times 100$

But BOD for $\log_{10} N = 0.35t + 2 \Rightarrow N = 10^{0.35t} \times 10^{2}$ o.e. such as $10^{0.35t} \times 100$

And BOD for $\log_{10} N = 0.35t + 2 \Rightarrow N = 10^{0.35t+2} \Rightarrow N = 100 \times 2.24^{t}$

A1: CSO $N = 100 \times 2.24^{t}$ with no errors following M1, dM1. Allow awrt 2.24

Condone $a = 100, b = 2.24$ following the award of M1 dM1

You should expect to see at least one correct line of working and no errors

Method 2: starting with $N = ab^{t}$

M1: Takes logs of both sides and applies one correct rule. E.g $N = ab^{t} \Rightarrow \log N = \log a + \log b^{t}$

Give BOD for those who write $N = ab^{t}$ followed by $\log N = \log a + t \log b$

dM1: And follows with $\log N = \log a + t \log b$ before setting

$\log a = 2$ and $\log b = 0.35$ and proceeding to values for *a* and *b*

A1: Fully shows and produces correct values following M1, dM1

CSO $N = 100 \times 2.24^{t}$ with no errors. Allow awrt 2.24

Condone $a = 100, b = 2.24$ following the award of M1 dM1

You should expect to see at least one correct line of working and no errors

Method 3: A simultaneous approach that finds values but does not show the result
M1: Sets up two simultaneous equations in *a* and *b*

E.g. Uses $t = 0, N = 100 \Rightarrow a = 100$ and $t = 1, N = 10^{2.35} \Rightarrow ab^1 = 223.9$

dM1: Solves to find values for *a* and *b*.
A0: Has not **shown** the result so cannot be awarded

Common error: Special Case

$\log_{10} N = 0.35t + 2 \Rightarrow N = 10^{0.35t} + 10^2$ so $a = 100$ and $b = 2.24$

Or even
$\log_{10} N = 0.35t + 2 \Rightarrow N = 10^{0.35t} + 10^2 \Rightarrow N = 100 \times 2.24^t$ so $a = 100$ and $b = 2.24$

Such work resulting in the **correct** values for *a* and *b* scores SC 100

(c) Full marks in part (c) can be scored following the SC

M1: Attempts $\frac{dN}{dt} = K \times "2.24"^5$ following through on their 2.24.

This cannot be scored from substituting *t* = 5 into their ab^t

A1: Calculates $\frac{dN}{dt} = 100 \times \ln 2.24 \times 2.24^5$ and achieves an answer in the range 4530 to 4550 (bacteria per hour)

Question Number	Scheme	Marks
5(a)	$\sin 3x = \sin(2x+x) = \sin 2x\cos x + \cos 2x\sin x$	M1
	$= (2\sin x\cos x)\cos x + (1-2\sin^2 x)\sin x$	dM1
	$= 2\sin x\cos^2 x + \sin x - 2\sin^3 x$	A1
	$= 2\sin x(1-\sin^2 x) + \sin x - 2\sin^3 x = 3\sin x - 4\sin^3 x$	A1
		(4)
(b)	$2\sin 3\theta = 5\sin 2\theta \Rightarrow 2(3\sin\theta - 4\sin^3\theta) = 10\sin\theta\cos\theta$	M1
	Divides or takes out $\sin\theta$ as a factor and uses $\sin^2\theta = 1-\cos^2\theta$ to set up and solve a 3TQ in $\cos\theta$ E.g. $\Rightarrow 6\sin\theta - 8\sin^3\theta = 10\sin\theta\cos\theta \Rightarrow 6 - 8(1-\cos^2\theta) = 10\cos\theta$ $\Rightarrow 4\cos^2\theta - 5\cos\theta - 1 = 0$ $\Rightarrow \cos\theta = \dfrac{5-\sqrt{41}}{8} = (-0.175...)$	dM1
	Any two of the following four answers $\sin\theta = 0 \Rightarrow \theta = 180°, 360°$ $\cos\theta = \dfrac{5-\sqrt{41}}{8} \Rightarrow \theta =$ awrt $100°$ or awrt $260°$	A1
	All of $180°, 360°$, awrt $100.1°$, awrt $259.9°$	A1
		(4)
		Total 8

(a)

M1: Attempts $\sin 3x = \sin(2x+x)$ and achieves $\pm\sin 2x\cos x \pm \cos 2x\sin x$. The $= \sin(2x+x)$ may be implied

dM1: Dependent upon the previous mark. It is for using $\sin 2x = 2\sin x\cos x$ and $\cos 2x = \pm 1 \pm 2\sin^2 x$, $\pm 1 \pm 2\cos^2 x$ or $\pm(\cos^2 x - \sin^2 x)$ to form an expression in just sin*x* and cos*x*.

A1: Fully correct AND expanded expression in just sin*x* and cos*x*

Examples of correct expanded expressions are;

$$2\sin x\cos^2 x + \sin x - 2\sin^3 x$$
$$2\sin x\cos^2 x + 2\sin x\cos^2 x - \sin x$$
$$2\sin x\cos^2 x + \sin x\cos^2 x - \sin^3 x$$
or $4\sin x\cos^2 x - \sin x$

Condone a missing *x* on a sin *x*, for example. Condone a slip in notation, e.g. writing $\sin x^3$ instead of $\sin^3 x$ as long as it is subsequently corrected.

A1: Proceeds correctly to $3\sin x - 4\sin^3 x$ using $\cos^2 x = 1 - \sin^2 x$

Penalise notational and bracketing errors on this mark only

E.g. Writing $\cos x^2$ for $\cos^2 x$, mixing variables, missing *x* on a sin *x* etc

(b) Allow if solved in terms of another variable, say x

The question states hence or otherwise solve $2\sin 3\theta = 5\sin 2\theta$

In almost all cases they will use their part (a) and proceed by the 'hence' route.

'Otherwise' routes could start with earlier versions of $2\sin 3\theta$ such as $2\left(2\sin x\cos^2 x + \sin x - 2\sin^3 x\right)$

In almost all cases the main mark scheme provides a framework of how the marks are awarded

M1: Uses their part (a) and a double angle for $\sin 2\theta$ of the form $G\sin\theta\cos\theta$ to form an equation in $\sin\theta$ and $\cos\theta$. Look for $R\sin\theta + S\sin^3\theta = T\sin\theta\cos\theta$ o.e. Condone slips such as forgetting to $\times 2$

dM1: The mark is scored for dividing or taking out a factor of $\sin\theta$ and using $\sin^2\theta = \pm 1 \pm \cos^2\theta$ to set up and solve a 3TQ in $\cos\theta$. To solve you should expect to see a valid method used, leading to at least one value for $\cos\theta = k, -1 < k < 1, k \neq 0$

If a calculator is used to solve the 3TQ in $\cos\theta$ the value(s) must be correct for their equation.

Candidates cannot go straight from an equation e.g. $4\cos^2\theta - 5\cos\theta - 1 = 0$ to a solution e.g. $100.1°$ without sight of the value for $\cos\theta$ in between.

A1: Finds two angles of $180°, 360°$, awrt $100°$, awrt $260°$ following at least one previous M1

The $180°$ and/or $360°$ must be found via a correct method with $\sin\theta = 0$ being seen or implied and also produced from a factorised $P\sin\theta + Q\sin^3\theta = R\sin\theta\cos\theta$

Likewise, the $100°$ and $260°$ must have been achieved via solving a correct equation.

Allow this mark if the answers are given in radians. So, for two angles of $\pi, 2\pi$, awrt 1.75, awrt 4.54

A1: All of $180°, 360°$, awrt $100.1°$, awrt $259.9°$ with no extra values within the range following M1, dM1

There are no marks for answers appearing without valid method and working

There are other (usually more complicated) ways of attempting part (b) so you must look carefully through each response. If you think a response deserves credit and you are unsure of how to mark, please send to review.

Question Number	Scheme	Marks
6 (a)	$\text{gf}(2)=\text{g}(3)=3^2+5$	M1
	$= 14$	A1
		(2)
(b)	Correct attempt at inverse $y=6-\dfrac{21}{2x+3}\Rightarrow 2x+3=\dfrac{21}{6-y}\Rightarrow x=\left(\dfrac{21}{6-y}-3\right)\div 2$	M1
	$\text{f}^{-1}(x)=\dfrac{3x+3}{2(6-x)}$ or $\text{f}^{-1}(x)=\dfrac{21}{2(6-x)}-\dfrac{3}{2}$	A1
	$-1\leqslant x<6$	B1
		(3)
(c)	$\text{gg}(x)=126\Rightarrow\left(x^2+5\right)^2+5=126\Rightarrow x^2+5=\sqrt{121}$	M1
	$\Rightarrow x^2=\text{"}6\text{"}\Rightarrow x=(\pm)\sqrt{6}$	dM1
	$\Rightarrow x=\pm\sqrt{6}$	A1
		(3)
		Total 8

(a)

M1: Full attempt at $\text{gf}(2)$ applying the rules in the correct order

Look for one of

- An attempt at $6-\dfrac{21}{2\times 2+3}\to k$ followed by an attempt at k^2+5
- An attempt at substituting x = 2 into $\left(6-\dfrac{21}{2x+3}\right)^2+5$

A1: 14

Allow just 14 with no incorrect working for both marks

(b)

M1: For an attempt at the method of finding the inverse. It is the process you are scoring here.

Look for a process that starts with *y* as a function of *x* and proceeds to *x* as a function of just *y*.

Starting with $y=6-\dfrac{21}{2x+3}$, or possibly a changed function of the form $y=\dfrac{ax+b}{2x+3}$, which proceeds to a form x = function of **just** y. Some candidates may swap x and y first. E.g $x=6-\dfrac{21}{2y+3}$ and proceed to function y = function of just x which is fine.

A1: Correct inverse $\text{f}^{-1}(x)=\dfrac{3x+3}{2(6-x)}$ or $\text{f}^{-1}(x)=\dfrac{21}{2(6-x)}-\dfrac{3}{2}$ o.e. but condone y or f^{-1} for $\text{f}^{-1}(x)$

Allow other equivalents such as $y=\dfrac{-3x-3}{2(x-6)}$, $y=\dfrac{10.5}{(6-x)}-1.5$ and $y=\left(\dfrac{21}{6-x}-3\right)\div 2$

ISW following a correct answer

B1: Correct domain written in correct form. E.g. $-1\leqslant x<6$, $[-1, 6)$

(c)

M1: For an attempt to "undo" g once in an attempt to find the value of $g(x)$, x^2 or x^2+5.

This may be scored for any of the following methods (condoning slips)

- $\left(x^2+5\right)^2+5=126 \Rightarrow x^2+5=\sqrt{121}$

  $\left(x^2+5\right)^2+5=126 \Rightarrow x^4+10x^2-96=0 \Rightarrow \left(x^2-6\right)\left(x^2+16\right)=0 \Rightarrow x^2=...$
- Condone $\left(x^2+5\right)^2+5=126 \Rightarrow x^4+10x^2-96=0 \Rightarrow x=$ awrt 2.45 OR $x^2=6$
- $g(x)=\sqrt{126-5}$

Do not award this mark if the candidate believes that to find x you only need to undo g once.
So, answers like $x=11$ or $x=6$ will score 0,0,0 unless further work is seen

dM1: For a complete attempt to find the exact solution to the equation. Examples of this would include

- $\left(x^2+5\right)^2+5=126 \Rightarrow x^2+5=\sqrt{121} \Rightarrow x=\sqrt{\sqrt{121}\pm 5}$
- $\left(x^2+5\right)^2+5=126 \Rightarrow x^4+10x^2-96=0 \Rightarrow \left(x^2-6\right)\left(x^2+16\right)=0 \Rightarrow x^2=6 \Rightarrow x=\sqrt{6}$
- $\left(x^2+5\right)^2+5=126 \Rightarrow x=\sqrt{6}$ or $\left(x^2+5\right)^2+5=126 \Rightarrow x=-\sqrt{6}$
- $\left(x^2+5\right)^2+5=126 \Rightarrow x^4+10x^2-96=0 \Rightarrow x=\sqrt{6}$
- $x=\sqrt{'11'-5}$

A1: $x=\pm\sqrt{6}$ only. Ignore any reference to $\pm 4i$ if included.

Sight of $\pm\sqrt{6}$ without incorrect working scores all 3 marks

Question Number	Scheme	Marks
7 (a)	$\mathrm{f}(x)=x^3\sqrt{4x+7} \Rightarrow \mathrm{f}'(x)=3x^2\sqrt{4x+7}+2x^3(4x+7)^{-\frac{1}{2}}$	M1, A1
	$\Rightarrow \mathrm{f}'(x)=3x^2\sqrt{4x+7}+\dfrac{2x^3}{\sqrt{4x+7}}=\dfrac{3x^2(4x+7)+2x^3}{\sqrt{4x+7}}$	dM1
	$\Rightarrow \mathrm{f}'(x)=\dfrac{7x^2(2x+3)}{\sqrt{4x+7}}$	A1
		(4)
(b)	Substitutes $x=-\dfrac{3}{2}$ into $x^3\sqrt{4x+7} \Rightarrow y=...$	M1
	$\left(-\dfrac{3}{2},-\dfrac{27}{8}\right)$	A1
		(2)
(c)	Attempts $-4\times"-\dfrac{27}{8}"$	M1
	$y \leqslant \dfrac{27}{2}$	A1
		(2)
(d)	$(2,7)$	M1, A1
		(2)
		Total 10

(a)

M1: Attempts the product rule to achieve $\mathrm{f}'(x)=Px^2\sqrt{4x+7}+Qx^3(4x+7)^{-\frac{1}{2}}$ where P and Q are positive constants.

Other methods are possible so look at each one carefully.

For example: $\mathrm{f}(x)=x^3\sqrt{4x+7} \Rightarrow \mathrm{f}(x)=\sqrt{4x^7+7x^6} \Rightarrow \mathrm{f}'(x)=\dfrac{1}{2}\left(4x^7+7x^6\right)^{-\frac{1}{2}}\times\left(28x^6+42x^5\right)$

The same principles for the main mark scheme can be applied

A1: $\mathrm{f}'(x)=3x^2\sqrt{4x+7}+2x^3(4x+7)^{-\frac{1}{2}}$ o.e. which may be left unsimplified

dM1: Dependent upon previous M1. It is for "correctly" producing a single fraction with a denominator of $\sqrt{4x+7}$.

Look for $\mathrm{f}'(x)=Px^2\sqrt{4x+7}+Qx^3(4x+7)^{-\frac{1}{2}} \Rightarrow \dfrac{Px^2(4x+7)+Qx^3}{\sqrt{4x+7}}$ o.e

A1: $\mathrm{f}'(x)=\dfrac{7x^2(2x+3)}{\sqrt{4x+7}}$

……………………………………………………………………………………………………

It is possible to attempt this by WMA14 methods using $[\mathrm{f}(x)]^2=x^6(4x+7)=4x^7+7x^6$

Score

M1, A1: $2\mathrm{f}(x)\mathrm{f}'(x)=28x^6+42x^5$

dM1, A1: $\mathrm{f}'(x)=\dfrac{14x^5(2x+5)}{2\mathrm{f}(x)}=\dfrac{14x^5(2x+5)}{2x^3\sqrt{4x+7}}=\dfrac{7x^2(2x+5)}{\sqrt{4x+7}}$

……
………………….

(b)

M1: Attempts to substitute $x=-\frac{3}{2}$ into $x^3\sqrt{4x+7}$ to find the y value of the minimum point .

This may be implied by the correct y value.

It can be attempted even if part (a) is missing or incorrect

If they ignore the given form of $\mathrm{f}'(x)$ and use their own version, they must

- Set their $\mathrm{f}'(x)$ which must be of the form $Px^2\sqrt{4x+7}+Qx^3\left(4x+7\right)^{-\frac{1}{2}}=0$
- Solve $Px^2\sqrt{4x+7}+Qx^3\left(4x+7\right)^{-\frac{1}{2}}=0$ by multiplying by $\left(4x+7\right)^{\frac{1}{2}}$ o.e. to achieve a non-zero value for x
- Substitute this non zero value for x in into $x^3\sqrt{4x+7}$ to find y

A1: $\left(-\frac{3}{2},-\frac{27}{8}\right)$ which may be awarded separately as, for example, $x=-1.5,\ y=-3.375$

Ignore (0, 0) if also given.

(c)

M1: Attempts $\pm 4\times "-\frac{27}{8}"$ which may be seen within an inequality.

A1: $y\leqslant\frac{27}{2}$ which must be correctly simplified.

Allow it to be written in other correct forms such as $\mathrm{g}\leqslant\frac{27}{2}$, $\mathrm{g}(x)\leqslant\frac{27}{2}$ or $\left(-\infty,13.5\right]$

The original function is f so $\mathrm{f}(x)\leqslant\frac{27}{2}$ would be M1 A0

(d)

M1: For one correct coordinate. Look for $\left(2,...\right)$ or $\left(...,7\right)$. Allow for either $x=2$ or $y=7$

If the coordinates have been built up mark the final answer.

E.g. $\left(\frac{1}{2},\ \frac{3}{8}\right)\to\left(2,\ \frac{3}{8}\right)\to\left(2,15\right)\to\left(-6,15\right)$ would score M0 A0

A1: $\left(2,7\right)$. Allow $x=2$ and $y=7$ ISW after a correct answer

Question Number	Scheme	Marks
8 (a)	52 b.p.m.	B1
		(1)
(b)	32 b.p.m.	B1
		(1)
(c)	$\frac{dH}{dt} = -8\,e^{-0.2t} + 18\,e^{-0.9t}$	M1, A1
	Sets $-8\,e^{-0.2t} + 18\,e^{-0.9t} = 0 \Rightarrow 4e^{0.7t} = 9$	dM1
	$\Rightarrow 0.7t = \ln\frac{9}{4} \Rightarrow t = ...$	M1
	T = 1.158 (minutes)	A1
		(5)
(d)	$37 = 32 + 40\,e^{-0.2t} - 20\,e^{-0.9t} \Rightarrow e^{-0.2t} = \frac{1+4\,e^{-0.9t}}{8}$	M1
	$\Rightarrow e^{0.2t} = \frac{8}{1+4\,e^{-0.9t}} \Rightarrow t = 5\ln\left(\frac{8}{1+4\,e^{-0.9t}}\right)$	A1*
		(2)
(e)	$t_2 = 5\ln\left(\frac{8}{1+4e^{-0.9\times 10}}\right) = ...$	M1
	(t_2) = awrt 10.3947	A1
	(M)= 10.3955	A 1
		(3)
		Total 12

(a)
B1: 52 b.p.m. Units are not required. Check for answer given in the body of the question
(b)
B1: 32 b.p.m. . Units are not required. Check for answer given in the body of the question
(c)
M1: Achieves $\frac{dH}{dt} = P\,e^{-0.2t} + Qe^{-0.9t}$ where P and Q are constants

A1: $\frac{dH}{dt} = -8\,e^{-0.2t} + 18\,e^{-0.9t}$ but allow un-simplified versions

dM1: Sets $P\,e^{-0.2t} + Q\,e^{-0.9t} = 0 \Rightarrow me^{0.7t} = n$ or $me^{-0.7t} = n$ where $m \times n > 0$

M1: Solves an equation of the form $me^{\pm kt} = n$ where $m \times n > 0$ using correct ln work.
It must proceed from a correct form of the derivative (so the first M1).
Look for $me^{\pm kt} = n \Rightarrow e^{\pm kt} = \frac{n}{m} \Rightarrow \pm kt = \ln\left(\frac{n}{m}\right) \Rightarrow t = ...$

A1: T = awrt 1.158 minutes following the award of all previous marks. You can ignore units
…………………………………………………………………………………………

An alternative for the last three marks in part (c) are;

dM1: $Pe^{-0.9t} = Qe^{-0.2t} \Rightarrow \ln P - 0.9t = \ln Q - 0.2t$

M1: Collects terms in t and proceeds to t =… Under this method it is dependent upon both previous M's
A1: T = awrt 1.158 minutes following the award of all previous marks. Units are not required

……
There may be answers produced from a calculator. In cases such as this we are going to apply the rule 'answers without full working may not gain credit.'

$$\frac{dH}{dt} = -8\,e^{-0.2t} + 18\,e^{-0.9t} = 0 \Rightarrow t = 1.158$$ scores 1,1,0,0,0

……

(d)

M1: Sets $H = 37$ and proceeds to make $e^{\pm 0.2t}$ the subject

A1*: Proceeds to the given answer showing all necessary steps.

Note that this is a given answer so the steps must be logical without huge jumps

Acceptable proofs:

Example 1

$$37 = 32 + 40\,e^{-0.2t} - 20\,e^{-0.9t} \Rightarrow e^{-0.2t} = \frac{1+4\,e^{-0.9t}}{8} \Rightarrow -0.2t = \ln\left(\frac{1+4\,e^{-0.9t}}{8}\right)$$

$$\Rightarrow t = -5\ln\left(\frac{1+4\,e^{-0.9t}}{8}\right)$$

$$\Rightarrow t = 5\ln\left(\frac{8}{1+4\,e^{-0.9t}}\right)$$

Example 2

$$37 = 32+40\,e^{-0.2t} - 20\,e^{-0.9t} \Rightarrow e^{-0.2t} = \frac{1+4\,e^{-0.9t}}{8} \Rightarrow e^{0.2t} = \frac{8}{1+4\,e^{-0.9t}} \Rightarrow 0.2t = \ln\left(\frac{8}{1+4\,e^{-0.9t}}\right) \Rightarrow t = 5\ln\left(\frac{8}{1+4\,e^{-0.9t}}\right)$$

Unacceptable proof:

$$37 = 32 + 40\,e^{-0.2t} - 20\,e^{-0.9t} \Rightarrow e^{-0.2t} = \frac{1+4\,e^{-0.9t}}{8} \Rightarrow -0.2t = \ln\left(\frac{1+4\,e^{-0.9t}}{8}\right)$$

$$\Rightarrow t = 5\ln\left(\frac{8}{1+4\,e^{-0.9t}}\right)$$

They need to deal with 0.2 and $-$sign in separate steps

……

Alt (d) lns can be taken earlier. The M1 is scored for making 0.2 t the subject. See below

$$37 = 32 + 40\,e^{-0.2t} - 20\,e^{-0.9t}$$

$$\Rightarrow 40e^{-0.2t} = 5 + 20\,e^{-0.9t} \Rightarrow \ln 40 - 0.2t = \ln\left(5+20\,e^{-0.9t}\right) \Rightarrow 0.2t = \ln 40 - \ln\left(5+20\,e^{-0.9t}\right)$$

We would need to see three further steps. E.g.

$$0.2t = \ln\left(\frac{40}{5+20\,e^{-0.9t}}\right) \Rightarrow t = 5\ln\left(\frac{40}{5+20\,e^{-0.9t}}\right) \Rightarrow t = 5\ln\left(\frac{8}{1+4\,e^{-0.9t}}\right)$$

……..

(e)

M1: Uses the iteration formula once. Allow for the embedded 10 leading to a value or sight of t_2 = awrt 10.4

A1: (t_2) = awrt 10.3947

A1: (M) = 10.3955 following some evidence of previous iterations, e.g. sight of

$t_2 = 5\ln\left(\frac{8}{1+4e^{-0.9\times10}}\right)$ = awrt 10.4..... This is NOT awrt so just 10.3955 here

Question Number	Scheme	Marks
9 (a)	$f'(x) = \frac{(2x+1)\times(12x+4)-2(6x^2+4x-2)}{(2x+1)^2}$	M1 A1
	$= \frac{12x^2+12x+8}{(2x+1)^2}$ o.e	A1
		(3)
(b)	At $x=2 \Rightarrow f'(x) = \frac{12\times2^2+12\times2+8}{(2\times2+1)^2} = \left(\frac{16}{5}\right)$	M1
	Full method of normal $y-6=-\frac{5}{16}(x-2)$	dM1
	$16y-96=-5x+10 \Rightarrow 16y+5x=106$ *	A1*
		(3)
(c)	Any correct value $A=3,\ B=\frac{1}{2}$ or $D=-\frac{5}{2}$	B1
	$6x^2+4x-2=(Ax+B)(2x+1)+D \Rightarrow$ Values of A, B and D	M1
	$3x+\frac{1}{2}+\frac{-5/2}{2x+1}$ o.e	A1
		(3)
(d)	$\int 3x+\frac{1}{2}+\frac{-5/2}{2x+1}\,dx = \frac{3}{2}x^2+\frac{1}{2}x-\frac{5}{4}\ln(2x+1)$	M1, A1 ft
	Area under curve = $\left[\frac{3}{2}x^2+\frac{1}{2}x-\frac{5}{4}\ln(2x+1)\right]_{\frac{1}{3}}^{2} = \left(\frac{3}{2}\times2^2+\frac{1}{2}\times2-\frac{5}{4}\ln(5)\right)-\left(\frac{3}{2}\times\left(\frac{1}{3}\right)^2+\frac{1}{2}\times\left(\frac{1}{3}\right)-\frac{5}{4}\ln\left(\frac{5}{3}\right)\right)$	dM1
	Area of R = $\int_{\frac{1}{3}}^{2} 3x+\frac{1}{2}+\frac{-5/2}{2x+1}dx+\frac{1}{2}\times6\times\left(\frac{106}{5}-2\right)$	M1
	Area of R = $\frac{964}{15}-\frac{5}{4}\ln3$	A1
		(5)
		Total 14

(a)

M1: Condoning slips, this is for an attempt using

- the quotient rule to obtain $f'(x) = \frac{(2x+1)\times(Ax+B)-C(6x^2+4x-2)}{(2x+1)^2}$ Must be '–' on the numerator
- the chain rule on $3x+A+\frac{B}{2x+1} \Rightarrow f'(x) = 3\pm D(2x+1)^{-2}$
- the product rule on $(6x^2+4x-2)(2x+1)^{-1} \Rightarrow f'(x) = (2x+1)^{-1}\times(Ax+B)-C(6x^2+4x-2)(2x+1)^{-2}$
  where A, B and C are > 0

A1: Correct (unsimplified) expression for $f'(x)$. Look for

- $f'(x)=\dfrac{(2x+1)\times(12x+4)-2\left(6x^2+4x-2\right)}{(2x+1)^2}$ via the quotient rule
- $f'(x)=(2x+1)^{-1}\times(12x+4)+\left(6x^2+4x-2\right)\times-2(2x+1)^{-2}$ via the product rule
- $3+5(2x+1)^{-2}$ following division. Condone an incorrect value for *A*.

A1: Simplified answers such as $\dfrac{4\left(3x^2+3x+2\right)}{(2x+1)^2}$, $\dfrac{12x^2+12x+8}{4x^2+4x+1}$ or $3+5(2x+1)^{-2}$ following correct *A*.

ISW following a correct answer

(b)

M1: Substitutes $x=2$ into their attempt at $f'(x)$.

If $f'(x)$ is incorrect then look for correctly embedded 2's or a correct value.

dM1: Full method for finding the equation of the normal at $(2, 6)$. Look for $y-6=-\dfrac{1}{f'(2)}(x-2)$

A1*: $y-6=-\dfrac{5}{16}(x-2)$ o.e with at least one correct intermediate line leading to $16y+5x=106$

If $y=mx+c$ is used look for $6=-\dfrac{5}{16}\times2+c\Rightarrow c=\dfrac{106}{16}$ You can allow $y=-\dfrac{5}{16}x+\dfrac{106}{16}$ going straight to the given answer. ISW following sight of correct answer. It must follow a correct derivative in (a)

(c)

B1: Any correct value for $A=3,\ B=\dfrac{1}{2}\ or\ D=-\dfrac{5}{2}$.

They may be embedded within an expression e.g. 3*x* in the quotient of a division sum

M1: Correct method for finding all three constants.

Via comparison/inspection look for $6x^2+4x-2=(Ax+B)(2x+1)+D\Rightarrow$ Values of *A*, *B* and *D*

Via division look for a linear quotient and a constant remainder

A1: $3x+\dfrac{1}{2}+\dfrac{-5/2}{2x+1}$ o.e such as $3x+\dfrac{1}{2}-\dfrac{5}{2(2x+1)}$ which may be seen within an integral in (d).

It is not just for values of *A*, *B* and *D*.

ISW following a correct answer (e.g. they may go on to $\times2$ but then the A1 ft in (d) would be unavailable)

(d)

M1: Attempts to integrate their $Ax+B+\dfrac{D}{2x+1}$. Look for two terms in the correct form, one of which must be the ln term. Award for $...x^2+....+k\ln(2x+1)$ or $...+...x+k\ln(2x+1)$

Also note that $k\ln(4x+2)$ and any multiple of $(2x+1)$ within the ln is also correct

A1ft: $\dfrac{3}{2}x^2+\dfrac{1}{2}x-\dfrac{5}{4}\ln(2x+1)$ o.e. but follow through on their values of *A*, *B* and *D*

dM1: A correct method of finding the area under the curve using limits of $\dfrac{1}{3}$ and 2 either way around.

Dependent upon the previous M mark. Award if the intention is clear.

M1: For a correct method of finding the area of *R*. It is scored for adding the area under the curve to the correct calculation for the area of the triangle. The integration of f(*x*) need not be correct but the limits must be applied the correct way around. The area of the triangle may be done by integration. Look for

$$\int_{\frac{1}{3}}^{2} f(x)\,dx+\frac{1}{2}\times 6\times\left(\frac{106}{5}-2\right)=\left[I(x)\right]_{\frac{1}{3}}^{2}+\frac{1}{2}\times 6\times\left(\frac{106}{5}-2\right)=I(2)-I\left(\frac{1}{3}\right)+\frac{1}{2}\times 6\times\left(\frac{106}{5}-2\right)=...$$

where $I(x)$ is their attempt at $\int f(x)\,dx$

A1: $\dfrac{964}{15}-\dfrac{5}{4}\ln 3$